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Seismic response reduction of irregular buildings using passive tuned mass dampers

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Abstract

This paper illustrates the practical considerations and vibration control effectiveness of passive tuned mass dampers (PTMDs) for irregular buildings, modelled as multi-storey torsionally coupled shear buildings, under bi-directional horizontal earthquake excitations. The PTMD is designed to control the mode which makes most contribution to the largest response of the building. Its optimum installation location and moving direction are determined from the controlled mode shape values. The optimal system parameters of PTMD are then calculated by minimizing the mean-square modal displacement response ratio of controlled mode between the building with and without PTMD under earthquake excitation from critical direction. As two PTMDs are used to reduce both translational responses, this study arranges the two mass dampers to achieve the largest vibration reduction. Numerical and statistical results from a long and a square five-storey torsionally coupled buildings subjected to five real earthquakes from different incident angles verify that the proposed optimal PTMDs are able to reduce the building responses effectively.

Keywords: Earthquake vibration control; Torsionally coupled building; Tuned mass damper

1. Introduction

Through intensive research and development in recent years, the passive tuned mass damper (PTMD) has been accepted as an effective vibration control device for both new structures and existing structures to enhance their reliability against winds, earthquakes, and human activities [1–12]. PTMDs can be incorporated into an existing structure with less interference compared with other passive energy dissipation devices. Since 1971, many PTMDs have been successfully installed in high-rise buildings and towers in the world (for example, the Citicorp Center in New York City, the John Hancock Building in Boston, USA; the Sydney Tower in Sydney, Australia; the Crystal Tower Building in Osaka and many observatory towers in Japan) and reported to be able to reduce wind-induced vibrations significantly. The determination of optimal system parameters (i.e. the mass, damping and stiffness coefficients) of a PTMD to decrease structural vibrations induced by different types of excitations is now well established [13–17]. The effectiveness of a single PTMD is decreased by its detuning frequency and off-optimum damping. In more recent studies [18–24], multiple tuned mass dampers (MTMDs) with distributed natural frequencies near the fundamental frequency of the main structure were proposed to improve the vibration control effectiveness. Almost all of these studies considered the controlled structure as a single degree-of-freedom (SDOF) system with its fundamental modal properties to design the PTMD and MTMDs. However, a real building usually possesses a large number of degrees of freedom and is actually asymmetric to some degree even with a nominally symmetric plan. It will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Thus, the simplified SDOF system which ignores the structural lateral–torsional coupling and the PTMD effect on different modes could overestimate the control effectiveness of PTMD [23]. In addition, it is well known that the vibration control of structures using PTMD is mainly attributed to the suppression of controlled modal responses. The previous

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studies determined the system parameters of PTMD based on this general concept. However, it is found in this paper that the vibration control effectiveness of a PTMD depends not only on the controlled modal parameters of the primary structure but also on the installed location and moving direction of the PTMD as well as the earthquake direction. Therefore, for a torsionally coupled real structure, the previously simplified model may lead to incorrect design of PTMD and overestimation of its vibration control effectiveness.

This study deals with the optimal installed floor, planar position and moving direction of PTMDs for irregular buildings under incident horizontal earthquake excitations. The building is modelled as a multi-storey torsionally coupled shear building with one rotation and two translations for each floor. The critical seismic incident angle to certain DOF of the building is determined such that its mean-square response under random excitation is maximum. The PTMD is designed to control the mode which makes most contribution to the largest response of the building. Its optimum installation location and moving direction are determined from the mode shape values of controlled mode. The optimal system parameters of PTMD are then calculated by minimizing the mean-square modal displacement response ratio of controlled mode between the building with and without PTMD under earthquake excitation from critical direction. As two PTMDs which have the same total mass as one PTMD are used to reduce both translational responses, this study arranges the two mass dampers to achieve the largest vibration reduction. Performance of the proposed optimal PTMDs is demonstrated by numerical and statistical studies of a long and a square five-storey torsionally coupled buildings subjected to five real earthquakes from various incident angles.

2. Dynamic equation of building TMD systems

With reference to the building idealization consisting of rigid floors supported on massless axially inextensible columns and walls, the general torsionally coupled multistorey buildings as shown in Fig. 1 have the following features: (1) the principal axes of resistance for all the stories are identically oriented, along the x and y-axes shown; (2) the centers of mass of the floors do not lie on a vertical axis; (3) centers of resistance of the stories do not lie on a vertical axis, either, i.e. the static eccentricities at each storey are not equal; (4) all floors do not have the same radius of gyration r about the vertical axis through the center of mass; and (5) ratios of the three stiffness quantities—translational stiffness in x and y directions and torsional stiffness—for any storey are different.

For the above general torsionally coupled N-storey building, each floor has three degrees of freedom: x- and

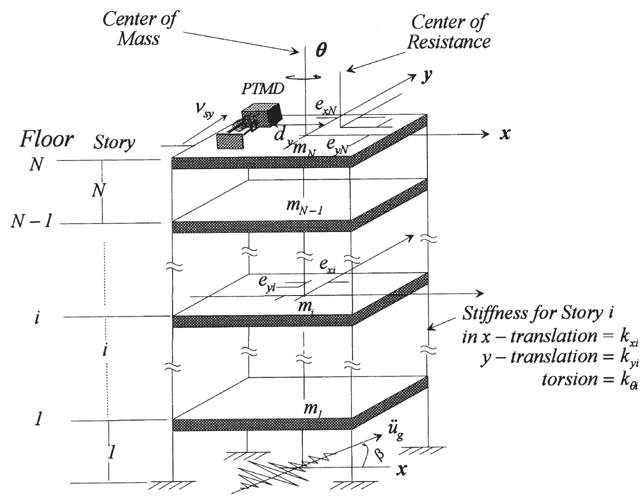

Fig. 1. N-storey general torsionally coupled building TMD system.

y-displacements, relative to the ground, of the center of mass and rotation about a vertical axis. For floor l, they are denoted by x_l, y_l and θ_l, respectively. Assumed that a SDOF PTMD of mass, m_{sy}, damping coefficient, c_{sy} and stiffness, k_{sy}, is installed at the lth floor with the distance of d_y to y-axis of lth floor, and moving in y direction. The dynamic equation of motion of the combined building TMD system under an incident horizontal earthquake excitation (incident angle β from x direction) can be written as

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0}^T & \boldsymbol{m}_{sy} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{u}}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C} & \boldsymbol{0} \\ \boldsymbol{0}^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{u}}_{sy} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0}^T & 0 \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{u}_{sy} \end{Bmatrix}$$
$$+ \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ [k_{sy}y_l + k_{sy}\upsilon_y r_l\theta_l - k_{sy}u_{sy}]_{(3l-1)th\ \mathrm{row}} \\ [k_{sy}\upsilon_y y_1 + k_{sy}\upsilon_y^2 r_l\theta_l - k_{sy}\upsilon_y u_{sy}]_{(3l)th\ \mathrm{row}} \\ 0 \\ \vdots \\ 0 \\ [k_{sy}(u_{sy} - y_l - \upsilon_y r_l\theta_l)]_{(3N+1)th\ \mathrm{row}} \end{Bmatrix} \tag{1}$$
$$+ \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ [c_{sy}\dot{y}_l + c_{sy}\upsilon_y r_l\dot{\theta}_l - c_{sy}\dot{u}_{sy}]_{(3l-1)th\ \mathrm{row}} \\ [c_{sy}\upsilon_y\dot{y}_l + c_{sy}\upsilon_y^2 r_l\dot{\theta}_l - c_{sy}\upsilon_y\dot{u}_{sy}]_{(3l)th\ \mathrm{row}} \\ 0 \\ \vdots \\ 0 \\ [c_{sy}(\dot{u}_{sy} - \dot{y}_l - \upsilon_y r_l\dot{\theta}_l)]_{(3N+1)th\ \mathrm{row}} \end{Bmatrix}$$

$$= \begin{Bmatrix} -\boldsymbol{Mr}\ddot{u}_g \\ -m_{sy}\sin\beta\ddot{u}_g \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F} \\ f_s \end{Bmatrix}$$

In Eq. (1), $\boldsymbol{M} = diag(\boldsymbol{M}_1\ \boldsymbol{M}_2\ldots\ \boldsymbol{M}_l\ldots\ \boldsymbol{M}_{\mathrm{N}}) = 3N \times 3N$ mass matrix of building, $\boldsymbol{M}_l = diag\ (m_l\ m_l\ m_l) = 3 \times 3$ mass submatrix, m_l is the lumped mass of floor l. Similarly, $\boldsymbol{K} = 3N \times 3N$ stiffness matrix of building and expressed as

$$\boldsymbol{K} = \begin{bmatrix} \boldsymbol{K}_{1,1} & \boldsymbol{K}_{1,2} & & & & & \\ \boldsymbol{K}_{2,1} & \boldsymbol{K}_{2,2} & \boldsymbol{K}_{2,3} & & & & \\ & \cdot & \cdot & \cdot & & & \\ & & \boldsymbol{K}_{l,l-1} & \boldsymbol{K}_{l,l} & \boldsymbol{K}_{l,l+1} & & \\ & & & \cdot & \cdot & \cdot & \\ & & & & \boldsymbol{K}_{N-1,N-2} & \boldsymbol{K}_{N-1,N-1} & \boldsymbol{K}_{N-1,N} \\ & & & & & \boldsymbol{K}_{N,N-1} & \boldsymbol{K}_{N,N} \end{bmatrix}$$

in which

$$\boldsymbol{K}_{l,l-1} = \begin{bmatrix} -k_{x_l} & 0 & k_{x_l}e_{y_{l-1,l}}/r_{l-1} \\ 0 & -k_{y_l} & -k_{y_l}e_{x_{l-1,l}}/r_{l-1} \\ k_{x_l}e_{y_{l,l}}/r_l & -k_{y_l}e_{x_{l,l}}/r_l & (-k_{\theta_l} - k_{x_l}e_{y_{l,l}}e_{y_{l-1,l}} - k_{y_l}e_{x_{l,l}}e_{x_{l-1,l}})/r_{l-1}r_l \end{bmatrix}$$

$(l = 2 \sim N)$

$$\boldsymbol{K}_{l,l} = \begin{bmatrix} k_{x_l} + k_{x_{l+1}} & 0 & (-k_{x_l}e_{y_{l,l}} - k_{x_{l+1}}e_{y_{l,l+1}})/r_l \\ 0 & k_{y_l} + k_{y_{l+1}} & (k_{y_l}e_{x_{l,l}} + k_{y_{l+1}}e_{x_{l,l+1}}/r_l) \\ (-k_{x_l}e_{y_{l,l}} - k_{x_{l+1}}e_{y_{l,l+1}})/r_l & (k_{y_l}e_{x_{l,l}} + k_{y_{l+1}}e_{x_{l,l+1}})/r_l & (k_{\theta_l} + k_{\theta_{l+1}} + k_{x_l}e^2_{y_{l,l}} + k_{x_{l+1}}e^2_{y_{l,l+1}} + k_{y_l}e^2_{x_{l,l}} + k_{y_{l+1}}e^2_{x_{l,l+1}})/r_l^2 \end{bmatrix}$$

$(l = 1 \sim N - 1)$

$$\boldsymbol{K}_{l,l} = \begin{bmatrix} k_{x_l} & 0 & -k_{x_l}e_{y_{l,l}}/r_l \\ 0 & k_{y_l} & k_{y_l}e_{x_{l,l}}/r_l \\ -k_{x_l}e_{y_{l,l}}/r_l & k_{y_l}e_{x_{l,l}}/r_l & (k_{\theta_l} + k_{x_l}e^2_{y_{l,l}} + k_{y_l}e^2_{x_{l,l}})/r_l^2 \end{bmatrix}$$

$(l = N)$

$$\boldsymbol{K}_{l,l+1} = \begin{bmatrix} -k_{x_{l+1}} & 0 & k_{x_{l+1}}e_{y_{l+1,l+1}}/r_{l+1} \\ 0 & -k_{y_{l+1}} & -k_{y_{l+1}}e_{x_{l+1,l+1}}/r_{l+1} \\ k_{x_{l+1}}e_{y_{l,l+1}}/r_l & -k_{y_{l+1}}e_{x_{l,l+1}}/r_l & (-k_{\theta_{l+1}} - k_{x_{l+1}}e_{y_{l,l+1}}e_{y_{l+1,l+1}} - k_{y_{l+1}}e_{x_{l,l+1}}e_{x_{l+1,l+1}})/r_lr_{l+1} \end{bmatrix}$$

$(l = 1 \sim N - 1)$

are the stiffness submatrices, where k_{x_l},k_{y_l} and k_{θ_l} are translational and rotational stiffnesses of storey l; $e_{x_{l,l}}$ and $e_{x_{l,l+1}}$ denote the static eccentricites in x-axis at floor l with respect to storey l and $l + 1$, respectively; and r_l is the radius of gyration of floor l. $\boldsymbol{u}^T = [x_1\ y_1\ r_1\theta_1\ldots\ x_l\ y_l\ r_l\theta_l\ldots\ x_N\ y_N\ r_N\theta_N]^T$ and u_{sy} denotes the displacement vector of primary structure and the PTMD displacement relative to base, respectively; T is the matrix transpose operator. $\boldsymbol{r}^T = [\cos\beta\ \sin\beta\ 0\ \cos\beta\ \sin\beta\ 0\ldots]^T$ is the ground influence coefficient vector; $\ddot{u}_g$ represents the incident earthquake ground acceleration; and $\upsilon_y = d_y/r_l$. Assumed that $\boldsymbol{C}$ is a classical damping matrix, the equation of motion of ith mode of controlled structure is expressed as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \frac{1}{m_i^*}\sum_{k=1}^{3N}\phi_{k,i}F_k + \frac{\phi_{3l,i}}{m_i^*}[\upsilon_y m_{sy}(2\xi_{sy}\omega_{sy}\dot{\nu}_{sy} + \omega_{sy}^2\nu_{sy})] + \frac{\phi_{3l-1,i}}{m_i^*}[m_{sy}(2\xi_{sy}\omega_{sy}\dot{\nu}_{sy} + \omega_{sy}^2\nu_{sy})] \tag{2}$$

In Eq. (2), $\upsilon_{sy} = u_{sy} - (y_l + d_y\theta_l)$ is the displacement of PTMD relative to the lth floor, or say PTMD's stroke; m_i^* and η_i are the ith generalized modal mass and displacement; $\omega_{sy} = \sqrt{k_{sy}/m_{sy}}$ and $\xi_{sy} = c_{sy}/(2m_{sy}\omega_{sy})$ represent the natural frequency and damping ratio of PTMD, respectively. $\phi_{3l-l,i}$ denotes the $(3l - 1)$th element of ith mode shape $\boldsymbol{\phi}_i$. F_k is the kth element of force vector $\boldsymbol{F}$ in Eq. (1). Define

$$\mu_{iy} = (\phi_{3l-1,i} + \upsilon_y\phi_{3l,i})\frac{m_{sy}}{m_i^*} = \rho_{iy}(1 + \upsilon_y(\phi_{3l,i}/\phi_{3l-1,i})) \text{ and } F_i^* = \frac{1}{m_i^*}\sum_{k=1}^{3N}\phi_{k,i}F_k \tag{3}$$

where $\rho_{iy} = \phi_{3l-1,i}(m_{sy}/m_i^*)$ denotes the ith modal mass ratio of PTMD. Then, Eq. (2) can be rewritten as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i - \mu_{iy}(2\xi_{sy}\omega_{sy}\dot{\nu}_{sy}) + \omega_i^2\eta_i - \mu_{iy}(\omega_{sy}^2\nu_{sy}) = F_i^* \tag{4}$$

Similarly, the equation of motion of PTMD in Eq. (1) becomes

$$\left(\sum_{k=1}^{3N}\phi_{3l-1,k}\ddot{\eta}_k + \upsilon_y\sum_{k=1}^{3N}\phi_{3l,k}\ddot{\eta}_k + \ddot{\nu}_{sy}\right) + 2\xi_{sy}\omega_{sy}\dot{\nu}_{sy} + \omega_{sy}^2\nu_{sy} = f_s^* \tag{5}$$

where $f_s^* = f_s/m_{sy}$. Provided that the PTMD is designed to tune the ith mode of controlled structure, from Eqs. (4) and (5), the equations of motion for ith mode and PTMD are expressed in matrix form as

$$\begin{bmatrix} 1 & 0 \\ (\phi_{3l-1,i} + \upsilon_y\phi_{3l,i}) & 1 \end{bmatrix}\begin{Bmatrix}\ddot{\eta}_i \\ \ddot{\nu}_{sy}\end{Bmatrix} + \begin{bmatrix} 2\xi_i\omega_i & -\mu_{iy}(2\xi_{sy}\omega_{sy}) \\ 0 & 2\xi_{sy}\omega_{sy}\end{bmatrix}\begin{Bmatrix}\dot{\eta}_i \\ \dot{\nu}_{sy}\end{Bmatrix} + \begin{bmatrix}\omega_i^2 & -\mu_{iy}\omega_{sy}^2 \\ 0 & \omega_{sy}^2\end{bmatrix}\begin{Bmatrix}\eta_i \\ \nu_{sy}\end{Bmatrix} = -\begin{Bmatrix}\Gamma_i \\ \sin\beta\end{Bmatrix}\ddot{u}_g \tag{6}$$

where $\Gamma_i = (\boldsymbol{\phi}_i^T\boldsymbol{Mr})/(\boldsymbol{\phi}_i^T\boldsymbol{M}\boldsymbol{\phi}_i)$ is the ith modal participation factor. It has been proved that as the primary structure has no lateral–torsional coupling, Eq. (6) is reduced to the same form as that of previous studies [1,25]. For the case of primary structure without PTMD, its ith modal equation of motion is given as

$$\ddot{\eta}_i + 2\xi_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = -\Gamma_i\ddot{u}_g \tag{7}$$

The comparison of structural modal responses in Eqs. (6) and (7) leads to the determination of optimal PTMD's system parameters and vibration control effectiveness of PTMD.

3. Optimal system parameters of PTMDs

According to Eqs. (6) and (7), the optimal PTMD's parameters are determined by minimizing the mean-square displacement response ratio of the ith tuned mode (or say controlled mode), $R_{dE,i}$, between the structure with and without installation of PTMD under an incident horizontal earthquake excitation. $R_{dE,i}$ takes the form as

$$R_{dE,i} = \frac{E[\eta_i^2]_{TMD}}{E[\eta_i^2]_{NOTMD}} = \frac{A}{B} \tag{8}$$

in which

$$\begin{aligned}
A = {} & 4\xi_i\xi_{sy}^3 r_{fy}^3(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& (1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& + 4\xi_i^2\xi_{sy}^2 r_{fy}^4(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& (1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& + 4\xi_i^3\xi_{sy} r_{fy}^3(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& + 4\xi_i^2\xi_{sy}^2 r_{fy}^2(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& - \xi_i\xi_{sy} r_{fy}^3(\Gamma_i^2 - \rho_{iy}^2(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& (1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& - \xi_i\xi_{sy} r_{fy}^3(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& + \Gamma_i^2\xi_i\xi_{sy} r_{fy} + \xi_i^2 r_{fy}^2(\rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& + \xi_i\xi_{sy} r_{fy}^5(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& (1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2)^2 \\
& + \xi_i^2 r_{fy}^4(\Gamma_i + \rho_{iy}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i})))^2 \\
& (\rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2)
\end{aligned}$$

$$\begin{aligned}
B = {} & 4\Gamma_i^2\xi_i\xi_{sy}^3 r_{fy}^3(1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& + 4\Gamma_i^2\xi_i^2\xi_{sy}^2 r_{fy}^2 \\
& + \Gamma_i^2\xi_{sy}^2 r_{fy}^2(\rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& + 4\Gamma_i^2\xi_i^2\xi_{sy}^2 r_{fy}^4(1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2) \\
& + \Gamma_i^2\xi_i\xi_{sy} r_{fy} + 4\Gamma_i^2\xi_i^3\xi_{sy} r_{fy}^3 - 2\Gamma_i^2\xi_i\xi_{sy} r_{fy}^3 \\
& + \Gamma_i^2\xi_i^2 r_{fy}^4(\rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{31,i}/\phi_{3l-1,i}))^2)^2 \\
& + \Gamma_i^2\xi_i\xi_{sy} r_{fy}^5(1 + \rho_{iy}\phi_{3l-1,i}(1 + \nu_y(\phi_{3l,i}/\phi_{3l-1,i}))^2)
\end{aligned}$$

where $r_{fy} = \omega_{sy}/\omega_i$ is defined as the frequency ratio of PTMD to the controlled mode. Owing to the magnitude of elements, $\phi_{k,i}$, in mode shape vector $\boldsymbol{\phi}_i$ is relative, the value $R_{dE,i}$ depends on $\phi_{k,i}$. If $\boldsymbol{\phi}_i$ is selected to normalize the ith effective modal mass $M_i = (\sum_{k=1}^{3N}\phi_{k,i}m_k r_k)^2/\sum_{k=1}^{3N}\phi_{k,i}^2 m_k$, $R_{dE,i}$ becomes independent of $\phi_{k,i}$, and Γ_i is reduced to 1.0. Under this definition, the modal mass ratio ρ_{iy} in Eq. (3) has a definite physical meaning and its expression becomes $\rho_{iy} = (\phi_{3l-1,i}m_{sy})M_i$.

The value of $R_{dE,i}$ smaller than unity represents the attenuation of structural responses due to the presence of PTMDs. It can be seen from Eq. (8) that $R_{dE,i}$ equals to unity as $\upsilon_y = (-\phi_{3l-1,i}/\phi_{3l,i})$. That means no vibration reduction. This finding indicates the importance of the considerations of lateral–torsional coupling effect and the installation location of PTMDs.

It is also seen from Eq. (8) that $R_{dE,i}$ is a function of the controlled modal parameters (ξ_i and $\boldsymbol{\phi}_i$), the PTMD's system parameters (ρ_{iy}, ξ_{sy} and r_{fy}), the installed floor ($\phi_{3l-1,i}$, $\phi_{3l,i}$), moving direction and planar position (υ_y) of PTMD, as well as the seismic incident angle β. For an existing building, when the controlled modal parameters, the installed floor, planar position and moving direction of PTMD, and the seismic incident angle β are known or given (to be discussed later), the optimal PTMD's system parameters can be obtained by differentiating $R_{dE,i}$ with respect to ρ_{iy}, r_{fy} and ξ_{sy} equating to zero, respectively, to minimize $R_{dE,i}$. Their values may be found by solving the following simultaneous equations

$$\frac{\partial R_{dE,i}}{\partial \rho_{iy}} = 0,\ \frac{\partial R_{dE,i}}{\partial r_{fy}} = 0,\ \frac{\partial R_{dE,i}}{\partial \xi_{sy}} = 0 \tag{9}$$

It has been found by Lin and colleagues [16,25] that an optimal modal mass ratio, $(\rho_{iy})_{opt}$ exists, but is rarely used due to economic considerations. Hence, in general, we find out $(r_{fy},\xi_{sy})_{opt}$ for various values of ρ_{iy} and then search for $(\rho_{iy})_{opt}$. As mentioned in preceding sections, prior to the determination of the optimal PTMD's design parameters from Eq. (9), we must first determine (i) the controlled structural mode; (ii) the installed floor, moving direction and planar position of PTMD, and (iii) the critical seismic incident angle β_{cr}. These factors play very important roles in optimum design of PTMDs and their control efficacy.

3.1. Controlled modes

As seen in the theoretical development, it is obvious that a PTMD is optimally designed to control the mode which makes the most contribution to a specified response of the primary structure. For a torsionally coupled shear building, the first three modes are the most important to the translational and torsional responses of each floor. However, the translations in x and y directions have different dominant modes. It is even possible to reduce the dynamic responses of all degrees of freedom using one PTMD, but this PTMD will not be the optimal one to every degree of freedom. In general, the conventional design of a PTMD is to reduce the largest response, which may cause damage, of the primary structure. Therefore, the dominant mode to the largest structural response is selected as the controlled mode of PTMD.

3.2. Installed floor, moving direction and planar position of PTMD

It has been shown by Lin et al. [25] for planar buildings that the floor corresponding to the tip of controlled mode shape will be the optimum location for PTMD, because more response reduction can be achieved. Similarly, for a torsionally coupled building, the terms of $\phi_{3l-1,i}$ and υ_y in Eq. (6) clearly denote the installed floor l, moving direction y, or say DOF $(3l-1)$, and planar position d_y. Therefore, the optimum installation floor and planar position of PTMD can be determined by means of maximizing the absolute values of $(\phi_{3l-1,i}+\upsilon_y\phi_{3l,i})$ for moving in y direction or $(\phi_{3l-2,i}+\upsilon_x\phi_{3l,i})$ for moving in x direction. To achieve this, the following steps are suggested: (i) choose the floor which has the largest mode shape value in controlled mode as the installed floor; (ii) choose the degree of freedom of the largest response as the moving direction of PTMD; (iii) the absolute value of $(\phi_{3l-1,i}+\upsilon_y\phi_{3l,i})$ depends on the sign of translation ($\phi_{3l-1,i}$ or $\phi_{3l-2,i}$) and rotation ($\phi_{3l,i}$) mode shape values. When both mode shape values have the same signs, we choose the maximum positive value of υ_y or υ_x allowable in the installed floor. On the other hand, when $\phi_{3l-1,i}$ or $\phi_{3l-2,i}$ and $\phi_{3l,i}$ have opposite signs, we choose υ_y or υ_x to be the maximum negative value. Through the above steps, it is concluded that the greater the distance between PTMD and the mass center of the installed floor, the more vibration reduction is obtained.

3.3. Critical seismic incident angle

The dynamic responses of a torsionally coupled building also depend on the incident angle of earthquake excitation. To design optimal PTMDs, it is essential and necessary to find the critical seismic incident angle which induces the largest structural responses. In this paper, the critical seismic incident angle, β_{cr}, is determined such that the mean-square response of the desired controlled degree of freedom is maximum. For an N-storey existing building, when only the first n_{ID} modal parameters are known by modal parameter identification techniques [25], its mean-square displacement response in y direction of top floor is expressed as

$$E[y_N^2] = \int_0^\infty \Big| \sum_{k=1}^{n_{ID}} \frac{-1}{m_k^*} [(-\omega^2 + 2i\xi_k\omega_k\omega \tag{10}$$

$$+\omega_k^2)^{-1}\cdot(\boldsymbol{\phi}_k^T \boldsymbol{Mr})]\phi_{3N-1,k}\Big|^2 \sin\beta S_{\ddot{u}_g}(\omega)d\omega$$

where $S_{\ddot{u}_g}(\omega)$ is the power spectral density of earthquake input.

4. Optimal design of second PTMD

According to above design procedure for one PTMD, the response of controlled DOF is reduced. Since this PTMD is designed based on the dominant modal properties of this DOF and its corresponding seismic incident angle, its capability in reducing the responses of other DOFs under an earthquake from different angles should be further investigated. It is found in this paper that whether additional PTMDs are required depends upon the degree of coupling among DOFs. For instance, a square building with nearly equal stiffnesses and small static eccentricities in x and y directions has slight coupling in x and y responses even though both translational frequencies are close. One PTMD designed for reducing y responses is not able to decrease x responses if an earthquake is applied from the critical incident angle of x responses and vice versa. Under this circumstance, a second PTMD becomes necessary.

Assumed that a second PTMD with mass m_{sx}, damping c_{sx}, and stiffness k_{sx}, is also mounted at the lth floor of the N-storey torsionally coupled building to tune the jth mode and moving in x direction. The equations of motion for the jth mode and two PTMDs are expressed in matrix form as

$$\begin{bmatrix} 1 & 0 & 0 \\ (\phi_{3l-2,j}+\upsilon_x\phi_{3l,j}) & 1 & 0 \\ (\phi_{3l-1,j}+\upsilon_y\phi_{3l,j}) & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_j \\ \ddot{\nu}_{sx} \\ \ddot{\nu}_{sy} \end{Bmatrix} + \begin{bmatrix} 2\xi_j\omega_j & -\mu_{jx}(2\xi_{sx}\omega_{sx}) & -\mu_{jy}(2\xi_{sy}\omega_{sy}) \\ 0 & 2\xi_{sx}\omega_{sx} & 0 \\ 0 & 0 & 2\xi_{sy}\omega_{sy} \end{bmatrix} \begin{Bmatrix} \dot{\eta}_j \\ \dot{\nu}_{sx} \\ \dot{\nu}_{sy} \end{Bmatrix} + \begin{bmatrix} \omega_j^2 & -\mu_{jx}\omega_{sx}^2 & -\mu_{jy}\omega_{sy}^2 \\ 0 & \omega_{sx}^2 & 0 \\ 0 & 0 & \omega_{sy}^2 \end{bmatrix} \begin{Bmatrix} \eta_j \\ \nu_{sx} \\ \nu_{sy} \end{Bmatrix} = -\begin{Bmatrix} \Gamma_j \\ \cos\beta \\ \sin\beta \end{Bmatrix} \ddot{u}_g \tag{11}$$

By matrix partition, Eq. (11) is separated into the following two expressions

$$\begin{bmatrix} 1 & 0 \\ (\phi_{3l-2,j}+\upsilon_x\phi_{3l,j}) & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\eta}_j \\ \ddot{\nu}_{sx} \end{Bmatrix} + \begin{bmatrix} 2\xi_j\omega_j & -\mu_{jx}(2\xi_{sx}\omega_{sx}) \\ 0 & 2\xi_{sx}\omega_{sx} \end{bmatrix} \begin{Bmatrix} \dot{\eta}_j \\ \dot{\nu}_{sx} \end{Bmatrix} + \begin{Bmatrix} -\mu_{jy}(2\xi_{sy}\omega_{sy})\dot{\nu}_{sy} \\ 0 \end{Bmatrix} + \begin{bmatrix} \omega_j^2 & -\mu_{jx}\omega_{sx}^2 \\ 0 & \omega_{sx}^2 \end{bmatrix} \begin{Bmatrix} \eta_j \\ \nu_{sx} \end{Bmatrix} + \begin{Bmatrix} -\mu_{jy}\omega_{sy}^2\nu_{sy} \\ 0 \end{Bmatrix} = -\begin{Bmatrix} \Gamma_j \\ \cos\beta \end{Bmatrix} \ddot{u}_g \tag{12}$$

$$(\phi_{3l-1,j}+\upsilon_y\phi_{3l,j})\ddot{\eta}_j + \ddot{\nu}_{sy} + 2\xi_{sy}\omega_{sy}\dot{\nu}_{sy} + \omega_{sy}^2\nu_{sy} = -\sin\beta\ddot{u}_g \tag{13}$$

Taking Fourier transform of Eqs. (12) and (13) and substituting Eq. (13) into Eq. (12), we obtain the equation of motion of the jth mode and the second PTMD as

$$\begin{bmatrix} -\omega^2+2i\omega\xi_j\omega_j+\omega_j^2+\dfrac{\omega^2(-2i\omega\mu_{jy}\xi_{sy}\omega_{sy}-\mu_{jy}\omega_{sy}^2)}{-\omega^2+2i\omega\xi_{sy}\omega_{sy}+\omega_{sy}^2}(\phi_{3l-1,j}+\upsilon_y\phi_{3l,j}) & -2i\omega\mu_{jx}\xi_{sx}\omega_{sx}-\mu_{jx}\omega_{sx}^2 \\ -\omega^2(\phi_{3l-2,j}+\upsilon_x\phi_{3l,j}) & -\omega^2+2i\omega\xi_{sx}\omega_{sx}+\omega_{sx}^2 \end{bmatrix} \begin{Bmatrix} \eta_j(i\omega) \\ \nu_{sx}(i\omega) \end{Bmatrix} = -\begin{Bmatrix} \Gamma_j+\sin\beta\dfrac{-2i\omega\mu_{jy}\xi_{sy}\omega_{sy}-\mu_{jy}\omega_{sy}^2}{-\omega^2+2i\omega\xi_{sy}\omega_{sy}+\omega_{sy}^2} \\ \cos\beta \end{Bmatrix} \ddot{\boldsymbol{u}}_g(i\omega) \tag{14}$$

In Eq. (14), $\eta_j(i\omega)$ denotes the Fourier displacement response of the jth mode of a building with two PTMDs. As Eqs. (8)–(10), the optimal system parameters of the second PTMD are determined by minimizing the mean-square displacement response ratio of the jth mode, $R_{dE,j}$, between the building with two PTMDs and with one PTMD under an incident horizontal earthquake from the critical angle of x responses, $\beta_{cr,x}$.

5. Numerical verifications

Two five-storey torsionally coupled buildings are presented to demonstrate the new design procedure and vibration control effectiveness of proposed optimal PTMDs. The first building (B1) has relative weak stiffnesses in y direction compared with those in x direction for each floor such as a long building. The second building (B2) has nearly equal stiffnesses in x and y directions. Tables 1 and 2 list their physical system parameters and first three modal frequencies, damping ratios and mode shapes. It is seen that the modal orders are $y-\theta-x$ and $y-x-\theta$ for B1 and B2, respectively. In addition, B2 possesses close translational modes which will cause unnegligible interaction between two modes. The total mass ratio of either one PTMD or two PTMDs to building total mass, is set to be 2% in the following numerical examples.

5.1. First PTMD

Because y direction is weak for both buildings, y direction of top floor is the controlled DOF and the first mode is thus the controlled mode. According to Eq. (10), the critical seismic incident angles of top floor displacement in y direction for both buildings are found to be 96° and 91° as shown in Figs. 2 and 3. Following the proposed design concept, the first PTMD is installed at the top floor, moving in y direction to reduce the first modal displacement under earthquake from $\beta = 96°$ or 91°. Since $\phi_{14,1}$ and $\phi_{15,1}$ have different signs (i.e. $-$32.096 and 14.454 for B1 and $-$5.234 and 2.578 for B2), υ_y is chosen to be $-$1.25 which corresponds to $d_y = -10$ m in opposite side of the resistance center for both buildings. The optimal PTMD's system parameters are then calculated by Eq. (9) and listed in Table 3.

The variation of mean square displacement response

Table 1
The physical system parameters of B1 and B2

Building		Floor mass m (kg)	Storey stiffness			Storey eccentricity (m)		Radius of gyration (m)
			k_x (N/m)	k_y (N/m)	k_θ (N-m)	e_x	e_y	
B1	1F	2.0×10^5	9.0×10^7	5.4×10^7	4.5×10^9	2.0	1.0	8.0
	2F	1.8×10^5	8.0×10^7	4.8×10^7	4.2×10^9	2.0	1.0	8.0
	3F	1.6×10^5	7.0×10^7	4.2×10^7	3.9×10^9	2.0	1.0	8.0
	4F	1.4×10^5	6.0×10^7	3.6×10^7	3.6×10^9	2.0	1.0	8.0
	5F	1.2×10^5	5.0×10^7	3.0×10^7	3.3×10^9	2.0	1.0	8.0
B2	1F	2.0×10^5	5.4×10^7	5.3×10^7	4.5×10^9	1.0	1.0	8.0
	2F	1.9×10^5	5.0×10^7	4.9×10^7	4.2×10^9	1.0	1.0	8.0
	3F	1.8×10^5	4.5×10^7	4.4×10^7	3.9×10^9	1.0	1.0	8.0
	4F	1.7×10^5	4.0×10^7	3.9×10^7	3.5×10^9	1.0	1.0	8.0
	5F	1.6×10^5	3.5×10^7	3.5×10^7	3.2×10^9	1.0	1.0	8.0

Table 2
The first three modal properties of B1 and B2

	Mode shapes						ω (Hz)		ξ (%)	
	x direction		y direction		$r\theta$ direction					
Building	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2
Mode 1	1.00	1.000	− 7.894	− 1.298	3.892	0.659	0.769	0.730	2.00	2.00
	1.987	1.988	− 15.848	− 2.583	7.636	1.305				
	2.863	2.890	− 23.086	− 3.761	10.851	1.877				
	3.533	3.608	− 28.786	− 4.700	13.206	2.327				
	3.907	4.026	− 32.096	− 5.234	14.454	2.578				
Mode 2	1.000	1.000	0.599	0.803	1.186	0.073	0.980	0.758	2.00	2.00
	2.005	1.988	1.228	1.597	2.322	0.145				
	2.918	2.894	1.830	2.328	3.292	0.209				
	3.635	3.615	1.330	2.912	3.998	0.259				
	4.050	4.037	2.636	3.244	4.370	0.287				
Mode 3	1.000	1.000	− 0.211	− 0.934	− 0.844	− 3.566	1.105	0.910	2.04	2.04
	2.022	1.994	− 0.442	− 1.865	− 1.650	− 7.055				
	2.969	2.923	− 0.674	− 2.738	− 2.335	− 10.142				
	3.731	3.668	− 0.876	− 3.442	− 2.830	− 12.566				
	4.182	4.114	− 1.004	− 3.849	− 3.090	− 13.919				

Table 3
Optimal system parameters of first PTMD for B1 and B2

Building	Controlled mode	Mass ratio (%)	ξ_s (%)	γ_f	Installed floor	Moving direction	υ_y
B1	Mode 1	2	14.0	0.91	5F	y	− 1.25
B2	Mode 2	2	12.0	0.93	5F	y	− 1.25

Table 4
Reduction of peak and root-mean-square responses of B1 under five real earthquakes

Earthquake	Peak response			RMS response		
	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)
El Centro (1940)	6.39	23.10	14.24	1.05	4.96	2.45
	(3.93)	(15.75)	(8.37)	(0.71)	(2.69)	(1.29)
Taft (1952)	5.52	17.15	8.55	1.00	3.13	1.60
	(3.79)	(13.56)	(6.28)	(0.78)	(1.74)	(0.94)
San Fernando (1971)	2.21	6.31	4.59	0.37	0.95	0.65
	(1.62)	(5.46)	(3.18)	(0.29)	(0.76)	(0.51)
Mexico (1985)	30.54	77.61	52.44	7.10	19.02	11.60
	(11.50)	(40.37)	(29.78)	(5.16)	(8.83)	(6.96)
Kobe (1995)	6.54	22.17	11.55	1.32	4.88	2.46
	(5.02)	(15.85)	(7.94)	(0.94)	(2.36)	(1.24)

(•) Responses with one PTMD.

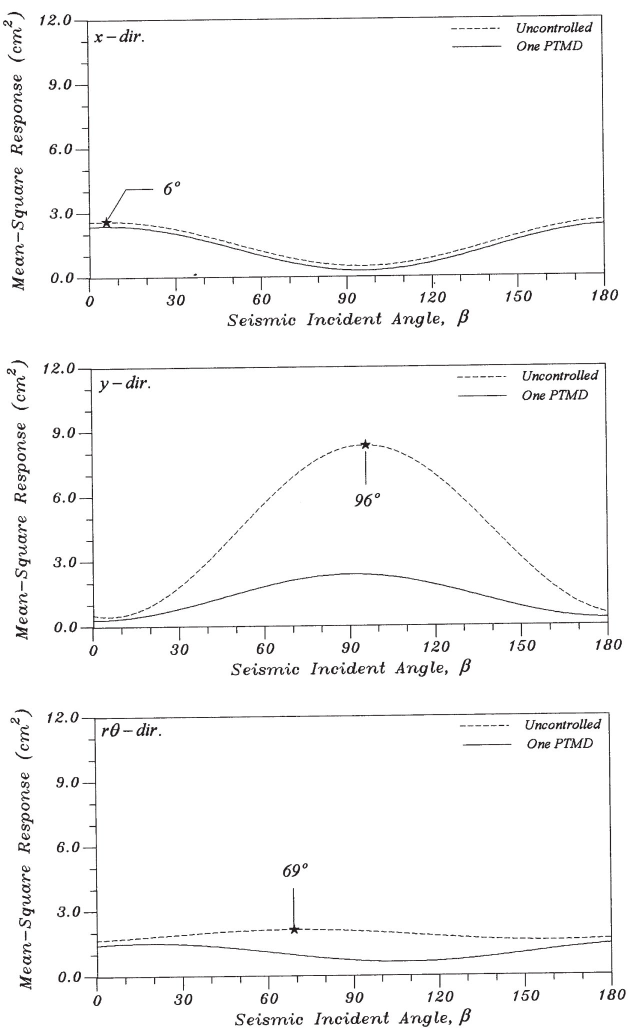

Fig. 2. Top floor mean-square displacement response of B1 with and without PTMD.

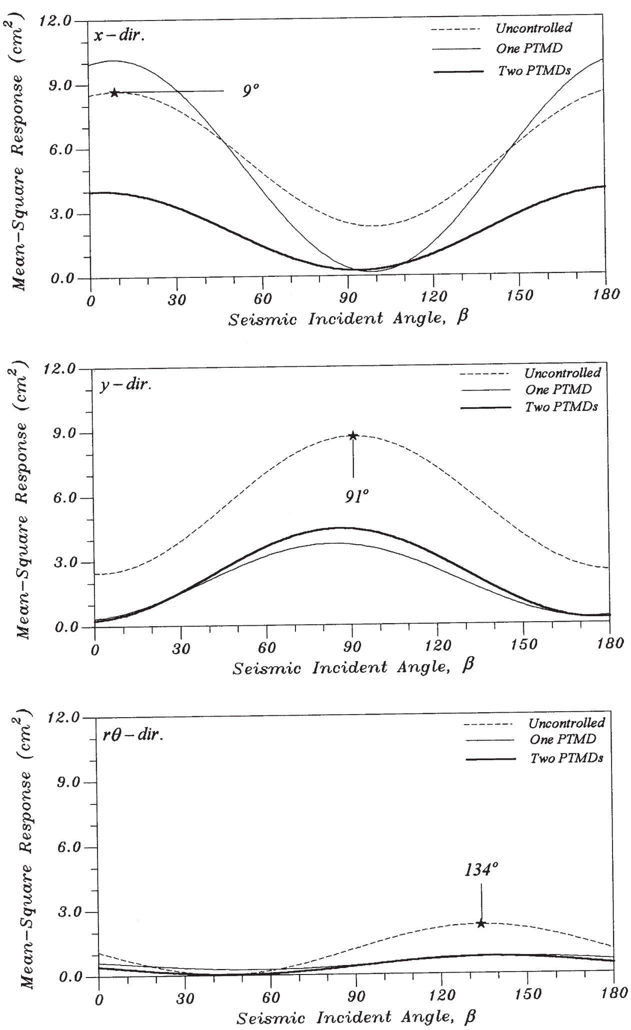

Fig. 3. Top floor mean-square displacement response of B2 with and without PTMD.

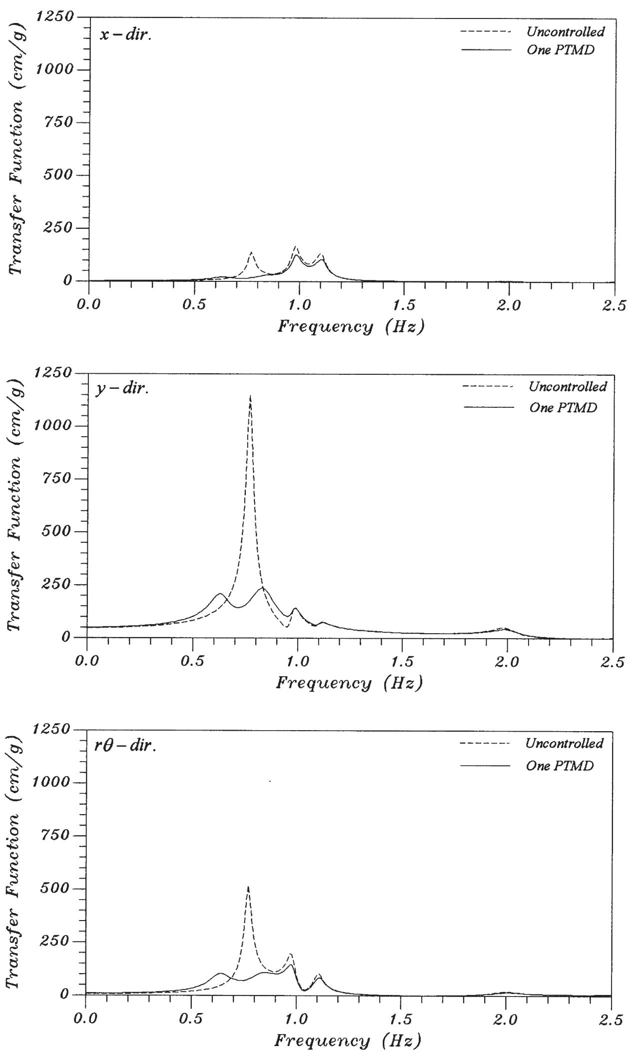

Fig. 4. Top floor displacement transfer function for B1 as β = 96°.

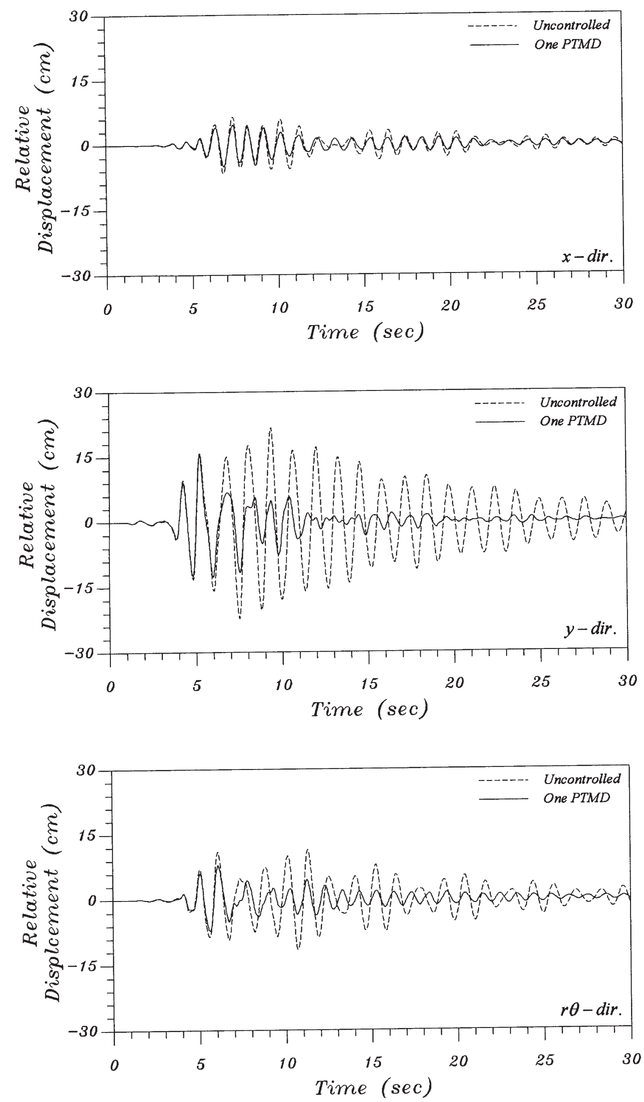

Fig. 5. Top floor displacement response of B1 under scaled-down Kobe earthquake from β = 96°.

of top floor with β is shown in Fig. 2 for B1 with and without PTMD. It is seen that all responses (particularly y response) are reduced for earthquakes from any incident angle. Thus, it is concluded that one optimal PTMD is adequate for the first type buildings. Fig. 4 depicts the transfer functions for $\beta = 96°$. It is apparent that the first modal amplitude in all three directional responses is suppressed significantly which agrees with the theoretical results. A statistical study is performed for B1 under five normalized (PGA = 0.3 g) real earthquakes, i.e. El Centro (1940), Taft (1952), San Fernando (1971), Mexico (1985) and Kobe (1995) from $\beta = 96°$. Table 4 lists the reductions of top floor peak and root-mean-square displacements with and without PTMD. Fig. 5 shows the time history displacement responses of the top floor under Kobe earthquake. As we expect, both peak and root-mean-square responses are reduced up to 40%. However, it is found in Fig. 3 that the top floor mean square displacement of B2 in x direction increases as β = (0–45)° and (145–180)°. This is attributed to the amplification of its dominant modal response (mode 2)

Table 5
Optimal system parameters of two PTMDs for B2

PTMD	Controlled mode	Mass ratio (%)	ξ_s (%)	γ_f	Installed floor	Moving direction	υ_x or υ_y
First	Mode 1	1	8.0	0.96	5F	y	− 1.25 (υ_y)
Second	Mode 2	1	6.0	0.99	5F	x	1.25 (υ_x)

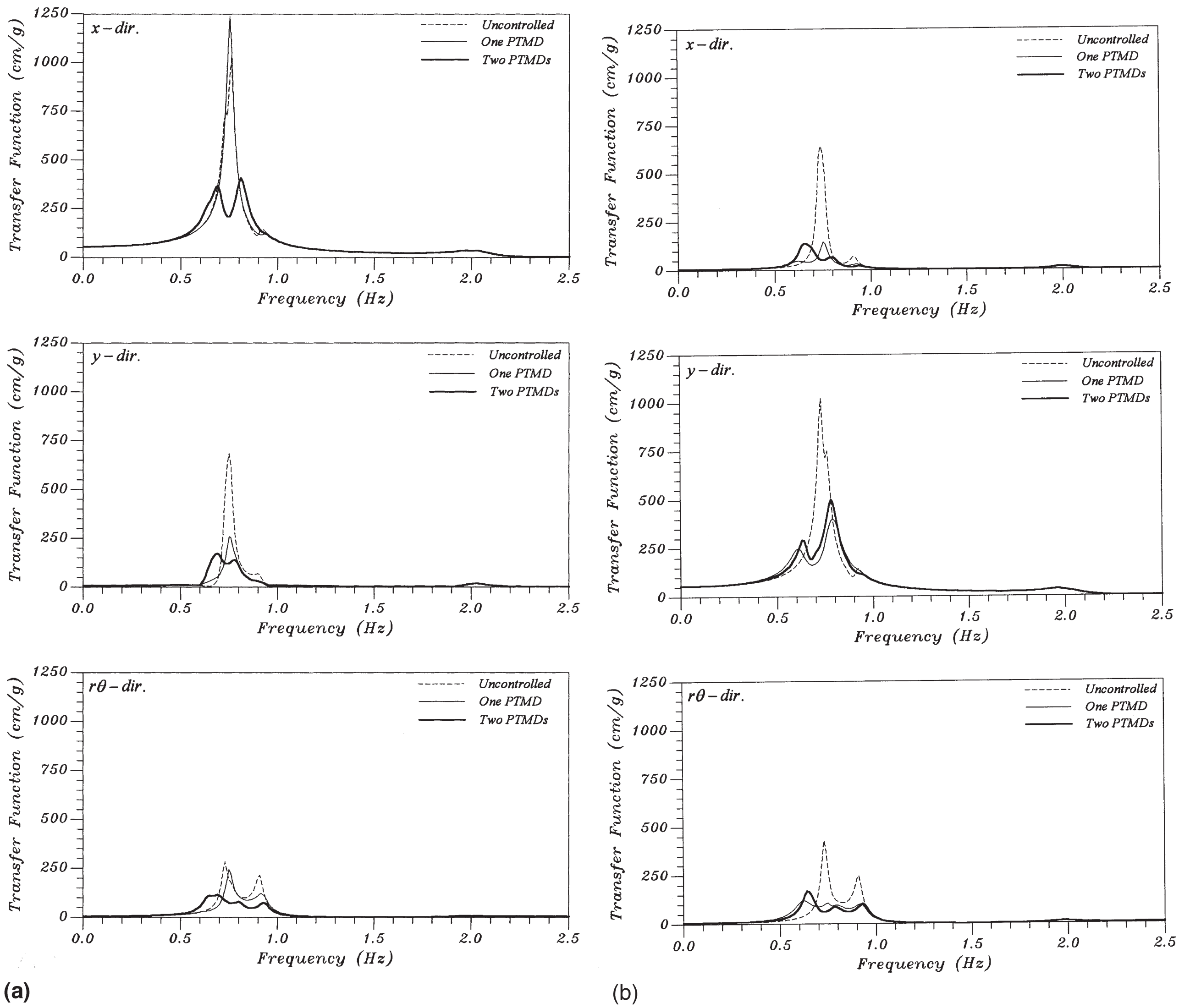

Fig. 6. (a) Top floor displacement transfer function for B2 as $\beta = 9°$. (b) Top floor displacement transfer function for B2 as $\beta = 91°$.

Table 6
Peak responses of B2 under scaled-down Kobe earthquake

	$\beta = 9° (\beta_{cr,x5})$			$\beta = 91° (\beta_{cr,y5})$		
	x_5 (cm)	y_5 (cm)	$(r\theta)_5$ (cm)	x_5 (cm)	y_5 (cm)	$(r0)_5$ (cm)
Uncontrolled	22.95	9.60	6.16	9.14	23.86	7.22
One PTMD	23.62	5.31	5.48	4.02	16.31	5.51
	(+ 3%)	(− 45%)	(− 11%)	(− 56%)	(− 32%)	(− 24%)
Two PTMDs	17.08	5.86	4.21	4.32	18.75	4.61
	(− 26%)	(− 39%)	(− 32%)	(− 53%)	(− 21%)	(− 36%)

after the installation of first PTMD. As observed in the preceding section, B2 has low coupling among three DOFs of each floor. Therefore, a second PTMD is required.

5.2. Second PTMD

To compare the vibration control effectiveness using one PTMD and two PTMDs, the same total mass of

PTMD is used for both cases. Through detailed numerical studies, we found that two PTMDs with equal mass will give the best control performance. The optimal locations and system parameters of two PTMDs for B2 are shown in Table 5. Fig. 6 illustrates the transfer functions of top floor displacement for B2 with one and two PTMDs under earthquakes from 9° ($\beta_{cr,x5}$) and 91° ($\beta_{cr,y5}$). The corresponding time history responses under scaled-down 1995 Kobe earthquake from $\beta = 9°$ are given in Fig. 7. Their peak responses are summarized in Table 6. The number in parenthesis (•) denotes the percentage of response reduction. It is seen that x_5 amplifies when only one PTMD is used in y direction. The fact of tremendous reduction of peak and root-mean-square responses again reveals the necessity and importance of the second PTMD.

6. Conclusions

This paper deals with the optimum installation location in plan and in elevation and moving direction of PTMDs for multi-storey torsionally coupled buildings under incident horizontal earthquake excitations. The optimal PTMD's system parameters are calculated by minimizing the mean-square total modal displacement response ratio of controlled mode between the building with and without PTMD under the earthquake excitation from critical direction. From theoretical developments and numerical results, the following conclusions are drawn: (1) the critical seismic incident angle is determined such that the mean-square response of the desired controlled DOF is maximum; (2) the dominant mode of desired controlled DOF is assigned as the controlled

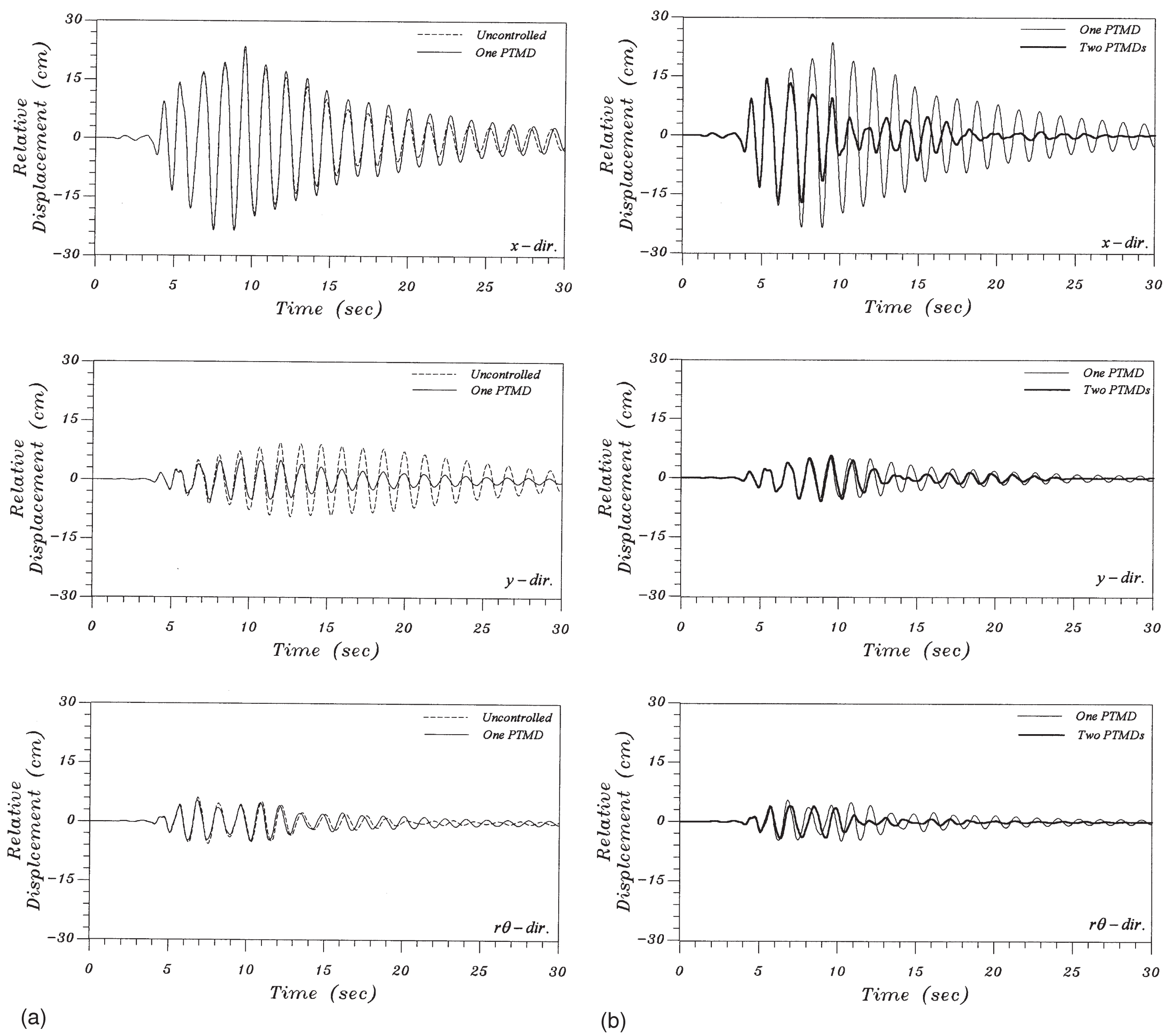

Fig. 7. Top floor displacement response of B2 under scaled-down Kobe earthquake from $\beta = 9°$.

mode of PTMD; (3) the floor corresponding to the tip of controlled mode shape is the optimum installed floor of PTMD; (4) the moving direction of PTMD is the same as the controlled DOF; (5) the greater the distance between PTMD and mass center of the installed floor, the more vibration reduction; (6) one PTMD is adequate in reducing both translations and rotation of long buildings under earthquakes from any incident angle. However, a second PTMD is required for buildings with nearly equal stiffness in x and y directions. Numerical and statistical results of a long and a square five-storey torsionally coupled building under five real earthquakes agree well with those of theoretical development.

Acknowledgements

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